EXPONENTIAL STABILITY OF STOCHASTIC HYBRID DIFFERENTIAL EQUATIONS WITH DELAYED IMPULSES

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ABSTRACT: *This paper focuses on the exponential stability of stochastic differential equations with delayed impulses and Markovian switching. A novel approach is developed to treat delayed impulses. Particularly, for a given stochastic differential equation and $p > 0$, a constant θ is introduced to summarize the contribution of delayed impulses on the pth moment exponential stability. In contrast to the progress in the literature, this paper provides a new criterion for the moment exponential stability.*

Keywords: *Stochastic differential equation, moment exponential stability, delayed impulses, Markovian switching.*

TÓM TẮT: *Bài viết này nghiên cứu tính ổn định mũ của các phương trình vi phân ngẫu nhiên với xung có trễ và bước chuyển Markov. Một phương pháp mới được phát triển để xử lí các xung có trễ. Cụ thể là, với mỗi phương trình vi phân ngẫu nhiên và p>0, một hằng số theta được dùng để đo lường sự đóng góp của các xung có trễ vào tính ổn định mũ theo moment cấp p của phương trình. Bài báo này đóng góp một tiêu chí mới cho tính ổn định mũ theo moment của lớp phương trình được nghiên cứu.*

Từ khóa: *Phương trình vi phân ngẫu nhiên, thời điểm ổn định mũ, xung có trễ, chuyển mạch Markovian.*

1. INTRODUCTION

This work focuses on the moment exponential stability of Markovian switching stochastic differential equations (MSDEs). Particularly, given by

$$\begin{aligned} dX(t) &= f(X(t), t, \alpha(t))dt \\ &+g(X(t), t, \alpha(t))dw(t),\ t \geq 0, \end{aligned} \tag{1}$$

with initial condition $(X(0), \alpha(0)) = (x_0, i_0) \in \mathbb{R}^d \times \mathcal{M}$, where $f(\cdot)$ and $g(\cdot)$ are suitable functions, $w(\cdot)$ is a Brownian motion, $\alpha(\cdot)$ is a finite state Markov chain. Suppose that Eq. (1) is not pth moment exponential stable (p is a positive constant). An effective approach is to introduce impulsive control to stabilize the given equation. In contrast to the formulations in [3, 7, 8] to mention just a few, we assume that at each time instant $t > 0$, one can only observe the state process $X(\cdot)$ with certain time delays. To be more specific, the dynamic system with impulses is given by

$$dX(t) = f(X(t), t, \alpha(t))dt$$

$$+ g(X(t), t, \alpha(t))dw(t), \quad t \geq 0,$$
$$X(t_k) = I_k\big(X(t_k - d_k), \alpha(t_k^-)\big),$$
$$k \in \mathbb{N}, \tag{2}$$

with initial condition $(X(0), \alpha(0)) = (x_0, i_0) \in \mathbb{R}^d \times \mathcal{M}$, where $I_k(\cdot)$ are suitable functions, $\{t_k\}_{k\geq 0}$ is a strictly increasing sequence of nonnegative numbers satisfying $t_0 = 0$ and $\lim_{k\to\infty} t_k = \infty$, $\{d_k\}_{k\in\mathbb{N}}$ is a sequence of nonnegative numbers satisfying $t_{k-1} < t_k - d_k$ for $k \in \mathbb{N}$. Here $X(t_k^-) = \lim_{t\to t_k^-} X(t)$ and $\alpha(t_k^-) = \lim_{t\to t_k^-} \alpha(t)$. The following question arises naturally: What are the contributions of the impulsive functions $\{I_k\}_{k\in\mathbb{N}}$ and the time delays $\{d_k\}_{k\in\mathbb{N}}$ to the pth moment exponential stability of Eq. (2)? The question motivates this paper.

Recently, impulsive systems with delayed impulses have received considerable attentions. To mention just a few of the recent development, we refer to [1, 2, 4, 5, 6] and references therein. In this paper, we employ novel approaches to the exponential stability of MSDEs with delayed impulses. For a given stochastic differential equation, a constant θ is introduced to summarize the contribution of delayed impulses on exponential stability. Then a stability criterion (Theorem 4) is established by using a comparison procedure.

The rest of the work is organized as follows. Section 2. begins with the problem formulation. Section 3. presents a new criterion for the moment exponential stability of impulsive MSDEs. Finally, the paper is concluded with several remarks in Section 3.

2. FORMULATION

We begin this section with the following notation.

Notation. Let $\mathbb{N} = \{1, 2, \dots\}$, $\mathbb{R}_+ = [0, \infty)$, $d, m_0 \in \mathbb{N}$, and $\mathcal{M} = \{1, 2, \dots, m_0\}$. For two real numbers c_1 and c_2, $c_1 \vee c_2$ denotes $\max\{c_1, c_2\}$. For $d_1, d_2 \in \mathbb{N}$, and a matrix $A \in \mathbb{R}^{d_1\times d_2}$, $A^\top$ denotes its transpose and $|A| = \sqrt{\text{tr}(A^\top A)}$ its trace norm. Particularly, for $x = (x_1, \dots, x_d)^\top \in \mathbb{R}^d$, $|x| = \big(\sum_{i=1}^d x_i^2\big)^{1/2}$ is its Euclidean norm. We work with a complete filtered probability space $(\Omega, \mathcal{F}, \mathbb{P}, \{\mathcal{F}_t\})$ with the filtration $\{\mathcal{F}_t\}$ satisfying the usual condition (i.e., it is right-continuous and $\mathcal{F}_0$ contains all the null sets). Assume that the Markov chain $\alpha(\cdot)$ and the $\widetilde{m}$-dimensional standard Brownian motion $w(\cdot)$ are defined on $(\Omega, \mathcal{F}, \mathbb{P}, \{\mathcal{F}_t\})$, where $\widetilde{m} \in \mathbb{N}$. Moreover, $\alpha(\cdot)$ and $w(\cdot)$ are $\{\mathcal{F}_t\}$-adapted and independent. Let $p > 0$ be a constant.

Suppose that $\alpha(\cdot)$ takes values in $\mathcal{M} = \{1, \dots, m_0\}$ with the generator $Q = (q_{ij}) \in \mathbb{R}^{m_0\times m_0}$. Let $f : \mathbb{R}^d \times \mathbb{R}_+ \times \mathcal{M} \to \mathbb{R}^d$, $g : \mathbb{R}^d \times \mathbb{R}_+ \times \mathcal{M} \to \mathbb{R}^{d\times\widetilde{m}}$, and $I_k : \mathbb{R}^d \times \mathcal{M} \to \mathbb{R}^d$ ($k \in \mathbb{N}$) be Borel measurable functions. Let $\{t_k\}_{k\geq 0}$ be a strictly increasing sequence of nonnegative numbers satisfying $t_0 = 0$ and $\lim_{k\to\infty} t_k = \infty$. Let $\{d_k\}_{k\in\mathbb{N}}$ be a sequence of positive numbers such that $t_{k-1} < t_k - d_k$ for each $k \in \mathbb{N}$.

Consider the impulsive MSDE

$$dX(t) = f\big(X(t), t, \alpha(t)\big)dt$$
$$+g\big(X(t), t, \alpha(t)\big)dw(t), t \geq 0, t \notin \{t_k\}_k,$$
$$X(t_k) = I_k(X(t_k - d_k), \alpha(t_k^-)),$$
$$k \in \mathbb{N}, \tag{3}$$

with initial condition $(x_0, i_0) \in \mathbb{R}^d \times \mathcal{M}$; that is,

$$X(0) = x_0, \;\; \alpha(0) = i_0 \in \mathcal{M}. \qquad (4)$$

We define an operator $\mathcal{L}$ associated with $(X(\cdot), \alpha(\cdot))$ as follows. Suppose $V : \mathbb{R}^d \times \mathbb{R}_+ \times \mathcal{M} \to \mathbb{R}$ is twice continuously differentiable in $x \in \mathbb{R}^d$ and continuously differentiable in $t \in \mathbb{R}_+$ for each $i \in \mathcal{M}$. Then

$$\begin{aligned}&\mathcal{L}V(x,t,i) = V_t(x,t,i)\\ &+V_x(x,t,i)f(x,t,i)\\ &+\frac{1}{2}\mathrm{tr}\big(g^\top(x,t,i)V_{xx}(x,t,i)g(x,t,i)\big)\\ &+\sum_{j\in\mathcal{M}} q_{ij}V(x,t,j),\end{aligned}$$

for $(x,t,i) \in \mathbb{R}^d \times \mathbb{R}_+ \times \mathcal{M}$, where $V_t(x,\cdot,i) = (\partial/\partial t)V(x,\cdot,i)$, $V_x(\cdot,t,i)$ and $V_{xx}(\cdot,t,i)$ denote the gradient and Hessian matrix of $V(\cdot,t,i)$, respectively. Throughout this work, we suppose the following assumption (A) holds.

(A) (a) We have $f(0,t,i) = g(0,t,i) = 0$ for all $(t,i) \in \mathbb{R}_+ \times \mathcal{M}$.

(b) For any real number $T > 0$ and $k \in \mathbb{N}$, there exists a positive number $M_{T,k}$ such that for all $t \in [0,T]$, $i \in \mathcal{M}$, and all $x, y \in \mathbb{R}^d$ with $|x|\vee|y| \le k$, $|f(x,t,i)-f(y,t,i)|^2 + |g(x,t,i)-g(y,t,i)|^2 \le M_{T,k}|x-y|^2$.

Also there exists a constant $M_{0,0} > 0$ such that

$x^\top f(x,t,i) \vee |g(x,t,i)|^2$

$\le M_{0,0}(1+|x|^2)$ for all $(x,t,i) \in \mathbb{R}^d \times \mathbb{R}_+ \times \mathcal{M}$.

(c) There exist positive constants M_k and $\widehat{M}_k$ for $k \in \mathbb{N}$ such that $M_k|x| \le \big|I_k(x,i)\big| \le \widehat{M}_k|x|$ for all $(x,i) \in \mathbb{R}^d \times \mathbb{R}^d \times \mathcal{M}, \;\; k \in \mathbb{N}$.

The existence and uniqueness theorem is given below. The proof is standard. Hence, we omit it for brevity.

Theorem 1. *Assume* (A). *Then for each* $(x_0, i_0) \in \mathbb{R}^d \times \mathcal{M}$*, Eq.* (3) *has a unique global solution* $(X^{x_0,i_0}(\cdot), \alpha^{i_0}(\cdot))$ *satisfying* (4). *Moreover, for each* $k \in \mathbb{N}$*,* $X^{x_0,i_0}(\cdot)$ *has continuous sample paths on the interval* $[t_{k-1}, t_k)$ *almost surely and* $\mathbb{E}\Big(\sup_{-r\le s\le T}|X^{x_0,i_0}(s)|^p\Big) < \infty$ *for any* $T > 0$.

Remark 2. Assumption (A)(c) guarantees the nonzero property; that is, $x_0 \neq 0$ implies

$$\mathbb{P}\Big(X^{x_0,i_0}(t) \neq 0 \quad \text{for all} \quad t \ge 0\Big) = 1.$$

We recall the definition of the pth moment exponential stability of impulsive MSDEs below.

Definition 3. Eq. (3) is said to be pth moment exponentially stable if there exist constants $K > 0$ and $\lambda > 0$ such that

$$\mathbb{E}|X^{x_0,i_0}(t)|^p \le Ke^{-\lambda t}|x_0|^p$$
$$\text{for } (x_0, i_0) \in \mathbb{R}^d \times \mathcal{M}, \;\; t \ge 0.$$

3. MAIN RESULTS

We are now in a position to state a criterion for the moment exponential stability.

Theorem 4. *Assume* (A). *Let* $V : \mathbb{R}^d \times \mathcal{M} \to \mathbb{R}$ *be a twice continuously differen-*

tiable on $\mathbb{R}^d \setminus \{0\}$ for each $i \in \mathcal{M}$ satisfying $r|x|^p \leq V(x,i)$ for $(x,i) \in \mathbb{R}^d \times \mathcal{M}$, where r is a positive number. Suppose that there exist numbers $\beta \in \mathbb{R}$, $\mu > 0$, and a sequence of nonnegative numbers $\{\rho_k\}_{k\in\mathbb{N}}$ such that

$$\begin{gathered}\mathcal{L}V(x,t,i) \leq \beta V(x,i),\\ (x,t,i) \in (\mathbb{R}^d \setminus \{0\}) \times \mathbb{R}_+ \times \mathcal{M},\end{gathered} \tag{5}$$

$$V(x,i) \leq \mu V(x,j), (x,i,j) \in \mathbb{R}^d \times \mathcal{M} \times \mathcal{M}, \tag{6}$$

and

$$\begin{gathered}V(I_k(x,i),i) \leq \rho_k V(x,i),\\ (x,i) \in \mathbb{R}^d \times \mathcal{M},\ \ k \in \mathbb{N}.\end{gathered} \tag{7}$$

Then the following assertions hold.

(a) *There exists a constant $K > 0$ such that*

$$\begin{gathered}\mathbb{E}|X^{x_0,i_0}(t)|^p \leq K|x_0|^p e^{\varphi(t)},\\ (x_0,i_0) \in \mathbb{R}^d \times \mathcal{M},\ \ t \geq 0,\end{gathered} \tag{8}$$

where

$$\varphi(t) := \beta t + \sum_{t_j \leq t} \ln\left(\rho_j \mu e^{-\beta d_j}\right), t \geq 0.$$

(b) *If*

$$\beta + \limsup_{t\to\infty} \frac{1}{t} \sum_{t_j \leq t} \ln(\rho_j \mu e^{-\beta d_j}) < 0, \tag{9}$$

then Eq. (3) *is pth moment exponentially stable.*

Proof. For notational simplicity, denote $X(\cdot) = X^{x_0,i_0}(\cdot)$ and $\alpha(\cdot) = \alpha^{i_0}(\cdot)$. Without loss of generality, suppose $x_0 \neq 0$. By Theorem 1, $\sup_{t\in[0,t_1)} \mathbb{E}V(X(t),\alpha(t)) < \infty$. Meanwhile, it follows from the definition of $\varphi(\cdot)$ that $\inf_{t\in[0,t_1)} e^{\varphi(t)} > 0$. Consequently, we can find a sufficiently large number $K_0 > 0$ such that

$$\mathbb{E}V(X(t),\alpha(t)) < K_1 e^{\varphi(t)},\ \ t \in [0,t_1), \tag{10}$$

where $K_1 = K_0|x_0|^p$.

(a) We prove by induction that for any $n \in \mathbb{N}$,

$$\mathbb{E}V(X(t),\alpha(t)) < K_1 e^{\varphi(t)},\ \ t \in [0,t_n). \tag{11}$$

In view of (10), (11) holds for $n = 1$. Now suppose (11) holds for $n \leq k$; that is,

$$\mathbb{E}V(X(t),\alpha(t)) < K_1 e^{\varphi(t)},\ \ t \in [0,t_k). \tag{12}$$

We proceed to show that (11) holds for $n = k+1$. Consider the real-valued functions $\Phi(\cdot)$ and $\Psi(\cdot)$ defined by

$$\begin{gathered}\Phi(t) = \mathbb{E}V(X(t),\alpha(t)), \Psi(t) = K_1 e^{\varphi(t)},\\ t \in [0,t_{k+1}).\end{gathered}$$

It is clear that $\Phi(t) < \Psi(t)$ for any $t \in [0,t_k)$. By (6), (7), and the properties of Markov chain $\alpha(\cdot)$, we have

$$\begin{aligned}&\mathbb{E}V(X(t_k),\alpha(t_k))\\ &= \mathbb{E}V\Big(I_k\big(X(t_k-d_k),\alpha(t_k^-)\big),\alpha(t_k^-)\Big)\\ &\leq \mathbb{E}\Big(\rho_k V\left(X(t_k-d_k),\alpha(t_k^-)\right)\Big)\\ &\leq \mathbb{E}\Big(\mu\rho_k V\left(X(t_k-d_k),\alpha(t_k-d_k)\right)\Big)\\ &= \mu\rho_k \mathbb{E}\Big(V\left(X(t_k-d_k),\alpha(t_k-d_k)\right)\Big),\end{aligned}$$

which together with (12) implies that

$$\begin{aligned}&\mathbb{E}V(X(t_k),\alpha(t_k)) < \rho_k\mu K_1 e^{\varphi(t_k-d_k)}\\ &= \rho_k\mu K_1 e^{\beta(t_k-d_k)+\sum_{t_j\leq t_k-d_k}\ln(\rho_j\mu e^{-\beta d_j})}\\ &= K_1 e^{\ln(\rho_k\mu e^{-\beta d_k})} e^{\beta t_k+\sum_{t_j<t_k}\ln(\rho_j\mu e^{-\beta d_j})}\\ &\leq K_1 e^{\varphi(t_k)};\end{aligned} \tag{13}$$

that is, $\Phi(t_k) < \Psi(t_k)$. By Theorem 1, $\Phi(\cdot)$ and $\Psi(\cdot)$ are continuous on $[t_k, t_{k+1})$. We claim that

$$\Phi(t) < \Psi(t), \;\; t \in (t_k, t_{k+1}). \tag{14}$$

If this statement were false, by the continuity of the functions $\Phi(\cdot)$ and $\Psi(\cdot)$, there would exist a number $t_* \in (t_k, t_{k+1})$ such that

$$\begin{aligned} &\Phi(t) < \Psi(t) \;\text{ for }\; t \in [t_k, t_*), \\ &\qquad\qquad \Phi(t_*) = \Psi(t_*). \end{aligned} \tag{15}$$

Fix $\lambda > |\beta|$. Consider the function $W(x,t,i) = e^{\lambda t} V(x,i)$ for $(x,t,i) \in \mathbb{R}^d \times [t_k, t_{k+1}) \times \mathcal{M}$. Applying the Itô formula and taking expectation of both sides of the resulting equation yield

$$\begin{aligned} &e^{\lambda t_*} \mathbb{E} V(X(t_*), \alpha(t_*)) \\ &= e^{\lambda t_k} \mathbb{E} V(X(t_k), \alpha(t_k)) \\ &\qquad + \mathbb{E} \int_{t_k}^{t_*} e^{\lambda s} \Big(\lambda V(X(s), \alpha(s)) \\ &+ \mathcal{L} V(X(s), s, \alpha(s)) \Big) ds. \end{aligned} \tag{16}$$

By (5),

$$\begin{aligned} &\mathbb{E} \int_{t_k}^{t_*} e^{\lambda s} \Big(\lambda V(X(s), \alpha(s)) \\ &+ \mathcal{L} V(X(s), s, \alpha(s)) \Big) ds \\ &\le \int_{t_k}^{t_*} e^{\lambda s} (\lambda + \beta) \mathbb{E} V(X(s), \alpha(s)) ds. \end{aligned} \tag{17}$$

In view of (15), we have

$$\mathbb{E} V(X(s), \alpha(s)) < K_1 e^{\varphi(s)}, \;\; s \in [t_k, t_*). \tag{18}$$

Combining the estimates in (17) and (18) yields

$$\begin{aligned} &\mathbb{E} \int_{t_k}^{t_*} e^{\lambda s} \Big(\lambda V(X(s), \alpha(s)) \\ &+ \mathcal{L} V(X(s), s, \alpha(s)) \Big) ds \\ &< K_1 \int_{t_k}^{t_*} e^{\lambda s + \varphi(s)} (\lambda + \beta) ds \\ &= K_1 e^{\sum_{t_j \le t_k} \ln(\rho_j \mu e^{-\beta d_j})} \\ &\int_{t_k}^{t_*} e^{(\lambda+\beta)s} (\lambda + \beta) ds \\ &= K_1 e^{\sum_{t_j \le t_k} \ln(\rho_j \mu e^{-\beta d_j})} \\ &\Big(e^{(\lambda+\beta)t_*} - e^{(\lambda+\beta)t_k} \Big). \end{aligned} \tag{19}$$

It follows from (13), (16), and (19) that

$$\begin{aligned} &e^{\lambda t_*} \mathbb{E} V(X(t_*), \alpha(t_*)) < K_1 e^{\lambda t_k + \varphi(t_k)} \\ &+ K_1 e^{\sum_{t_j \le t_k} \ln(\rho_j \mu e^{-\beta d_j})} \Big(e^{(\lambda+\beta)t_*} - e^{(\lambda+\beta)t_k} \Big) \\ &= K_1 e^{\sum_{t_j \le t_k} \ln(\rho_j \mu e^{-\beta d_j})} e^{(\lambda+\beta)t_*}. \end{aligned}$$

Hence

$$\begin{aligned} &\Phi(t_*) = \mathbb{E} V(X(t_*), \alpha(t_*)) \\ &< K_1 e^{\beta t_* + \sum_{t_j \le t_k} \ln(\rho_j \mu e^{-\beta d_j})} = \Psi(t_*), \end{aligned}$$

which contradicts the second statement in (15). As a result, (14) holds. Consequently,

$$\mathbb{E} V(X(t), \alpha(t)) < K_1 e^{\varphi(t)}, \;\; t \ge 0,$$

which implies (8).

(b) The conclusion follows from (8) and (9).
□

Remark 5. Under the conditions of Theorem 4,

$$\begin{aligned} &\limsup_{t\to\infty} \frac{1}{t} \ln \left(\mathbb{E} |X^{x_0, i_0}(t)|^p \right) \\ &\le \beta + \limsup_{t\to\infty} \frac{1}{t} \sum_{t_j \le t} \ln(\rho_j \mu e^{-\beta d_j}). \end{aligned} \tag{20}$$

The left hand side of (20) is known as the pth moment Lyapunov exponent of Eq. (3). The corresponding impulsive-free MSDE of Eq. (3) is

$$\begin{aligned} &dX(t) = f\big(X(t), t, \alpha(t)\big) dt \\ &+ g\big(X(t), t, \alpha(t)\big) dw(t), \;\; t \ge 0. \end{aligned} \tag{21}$$

By virtue of Theorem 4, the pth moment Lyapunov exponent of the impulsive-free MSDE (21) is not greater than β. The constant

$$\theta := \limsup_{t\to\infty} \frac{1}{t} \sum_{t_j \le t} \ln(\rho_j \mu e^{-\beta d_j})$$

can be regarded as the contribution of the impulses to the pth moment Lyapunov exponent of Eq. (3). It is readily seen that because of the delays of the impulses, the value of θ also depends on β. By Theorem 4, the pth moment Lyapunov exponent of the impulsive MSDE (3) is not greater than $\beta + \theta$. If $\theta < 0$, the impulses have positive effects on the pth moment exponential stability of Eq. (3). Particularly, if $\theta < 0$ and the magnitude of θ is sufficiently large so that $\beta + \theta < 0$, then Eq. (3) is pth moment exponentially stable.

4. CONCLUDING REMARKS

This work has focused on the moment exponential stability of MSDEs with delayed impulses. By using a new approach, we have developed a new criterion for the pth moment exponential stability. Problems of interest for future study include moment exponential stability of neutral stochastic functional differential equations with Markovian switching and other forms of impulsive perturbations.

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